Institute and Faculty of Actuaries

Loss Ratio Uncertainty

Andrew D Smith, University College Dublin
ansmith@ucd.ie

01 August 2018

Open up the Spreadsheet FitGamma03.xls

- Enter your name into cell C4 (you might need to enable editing)
- You now have your personal set of historic loss ratio data.
- The data depends on the last 3 letters of your name, so if you're a Smith like me, we'll have the same data.

Institute and Faculty of Actuaries

Your Loss Ratio Data

- These are loss ratios: claims divided by premiums, for 25 consecutive years.
- They might be on an underwriting year basis, or on an accident year basis.
  - It doesn't matter for our purposes.
- Assume all years are fully run off.
- Show of hands for average loss ratio.
- Show of hands for standard deviation of loss ratio.

Plot a Picture of Historic Loss Ratios against Time

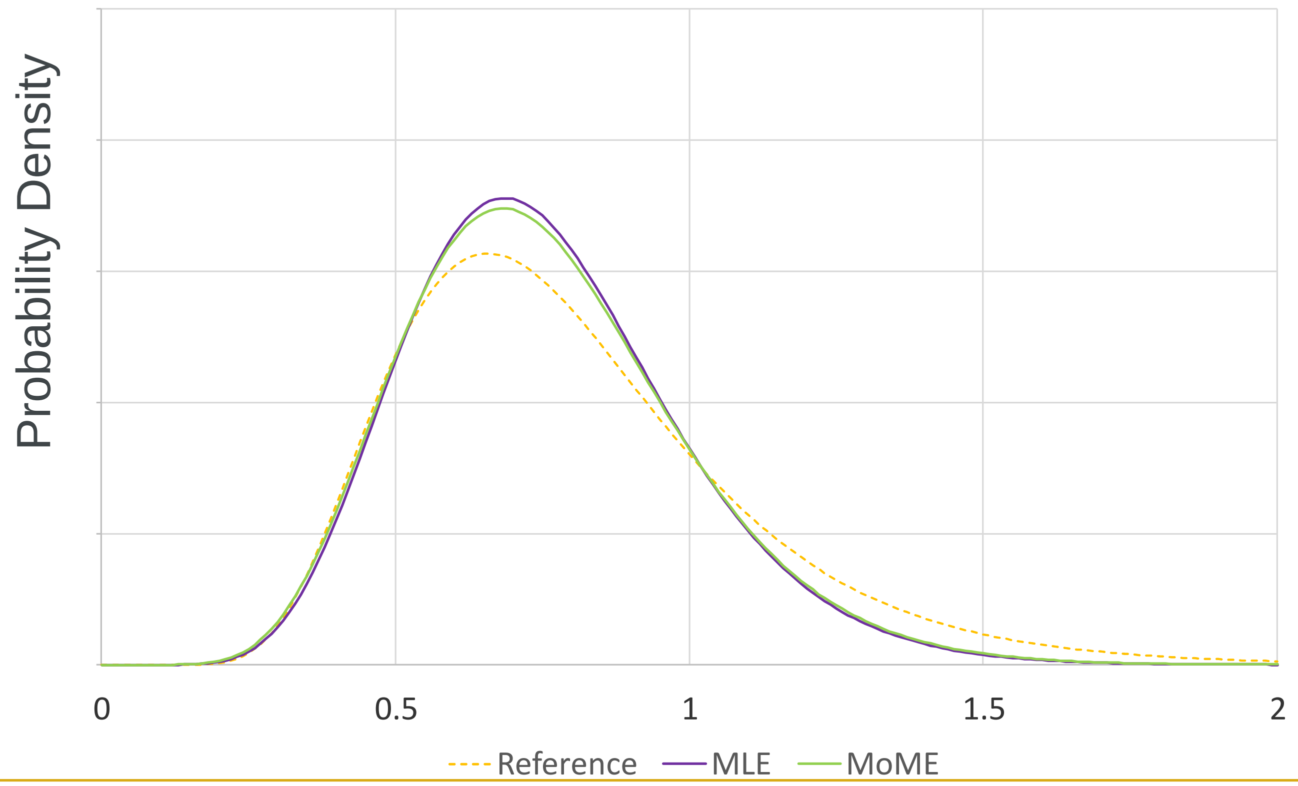
- What do you notice?
- Are they dependent?
- Is there a cycle?
- Can you even tell?

Institute and Faculty of Actuaries

Formula for Required Capital

- You are now in charge of an insurer's internal model, responsible for calculating the solvency capital requirement at a 99.5% confidence level.
  - The required capital will be expressed as a fraction of the premium.
  - You have to estimate the relevant fraction.
  - You only have 25 years' data, so fit a distribution and extrapolate to find the 99.5%-ile
- Choose a Gamma distribution and estimate the two parameters by
  - Maximum likelihood estimates (MLE)
  - Method of moments estimates (MoME)
- Gamma distributions have been widely used for loss ratio models, starting with Lloyds RBC model in the 1990s.

Fitted (Ersatz) Loss Ratio Distribution Comparison



Institute and Faculty of Actuaries

Comparing the Two Fits

- The mean of a gamma distribution is $\alpha\beta$ and the standard deviation is $\alpha^{1/2}\beta$.
- Who has higher mean loss ratio for MoME? For MLE?
- Who has higher standard deviation for MoME? For MLE?
- If you had to validate the model, you might test the fitted standard deviation against the data. Which method would this test favour?

Institute and Faculty of Actuaries

Goodness of Fit Tests

- Construction of P-P Plots
- This has:
  - On the X-axis, the fitted probability of a loss ratio ≤ *r*, for a range of values of *r*
  - On the Y-axis the observed proportion of observations ≤ *r*. As we have 25 observations, this goes up in steps of 4%.
- We are hoping for something close to a straight line if the distribution fits well.
- The acceptable deviations from a straight line are governed by the Kolmogorov-Smirnov (KS) distribution. Limits (2-sided 95%) shown in red.

Institute and Faculty of Actuaries

Did anyone fail the KS Test?

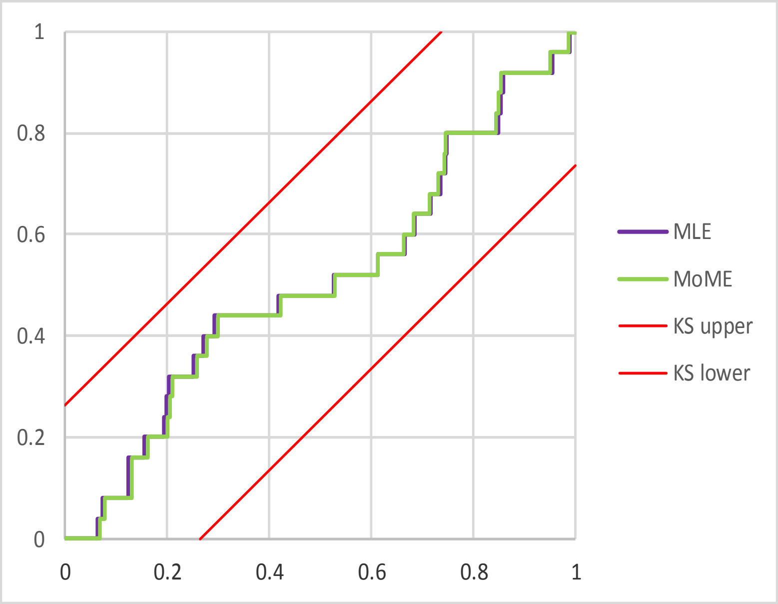
- No?
- That’s odd because out of 20 people we’d expect one failure.
- Alternative tests, usually with similar results, look at the area of PP deviations rather than the largest deviation. These give rise to Cramer-von Mises and Anderson-Darling tests.
- We could also look at QQ plots. On the Y-axis we show the observations sorted into increasing order and on the X-axis we show the corresponding quantiles of the fitted distribution. For a good fit, we are looking for a 45 degree line, but (unlike for PP) there is no general distribution for how large deviations could be.

Institute and Faculty of Actuaries

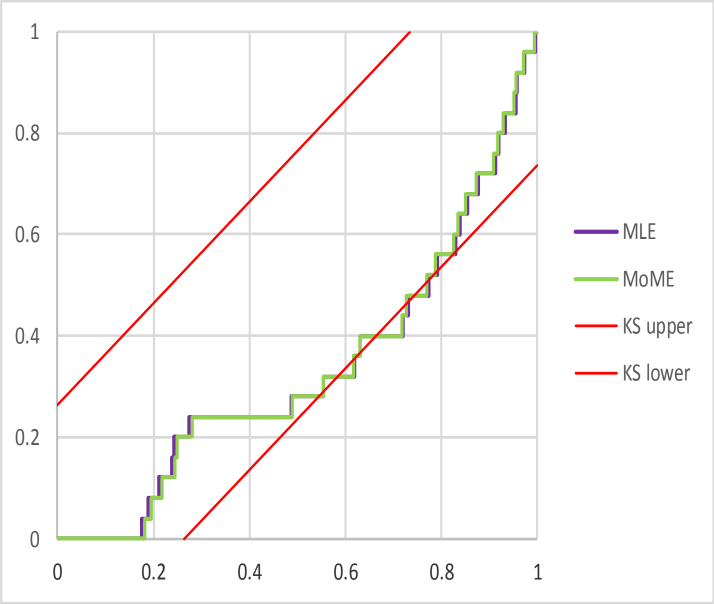
Rip van Winkel Experiment

- You fitted your model and then went to sleep for 25 years.
- You have 25 more data points.
- How now would you test your model?
- Back-testing a 1-in-10 event, you would expect 2.5 exceptions in 25 years.
- Raise your hand if you have an exception
  - at the 1-in-100 level,
  - or 1-in-1000?

Example P-P Plots: In-sample and Out-of-sample



In-sample



Out-of-sample

Institute and Faculty of Actuaries

Goodness of Fit Tests and the Lilliefors Effect

Context	Effect	H_0	Test size if unadjusted KS bounds used
Kolmogorov-Smirnov		Data is a random sample from the stated distribution.	5%
In-sample fit	Lilliefors	Data is a random sample from some gamma distribution.	< 5% As fitted mean = sample mean
Out-of-sample fit (Rip van Winkel)	Reverse Lilliefors	Past and future data are random samples from a common gamma distribution.	> 5%

- An abnormally low failure rate (the Lilliefors effect) is welcome for generating green lights in validation reports
- Should validators be concerned if failure rate < 5% ?

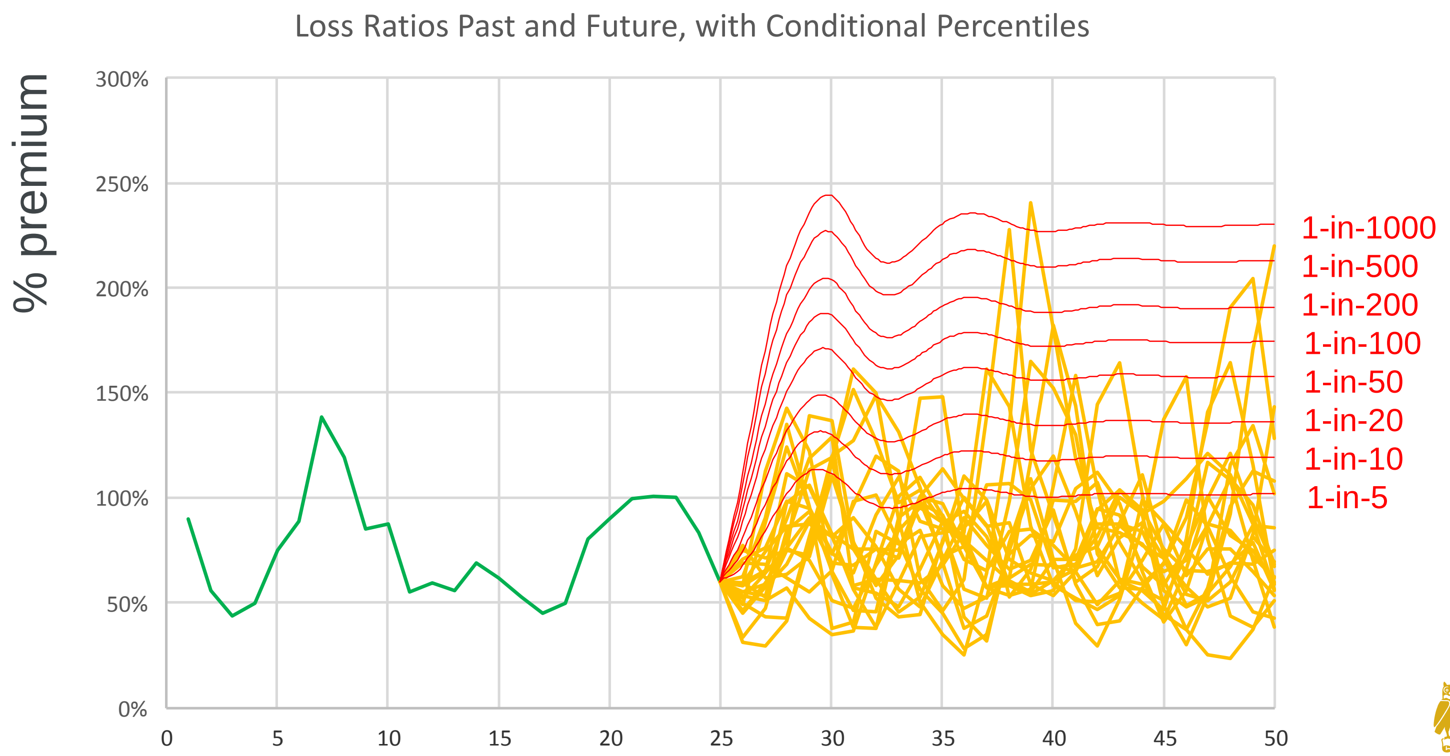
Institute and Faculty of Actuaries

The Bit we Can't Do in Real Life

- Given real data, we don't know the generating process, so we can't tell for sure how accurate the fitted model is.
- This loss ratio example used generated data from a *reference model,*
  - which in fact was the exponential of a normal AR2 time series.
- The models you have fitted – IID Gamma – are *ersatz models,*
  - ie substitutes for the reference model
- Our experiment has parameter error and model error.

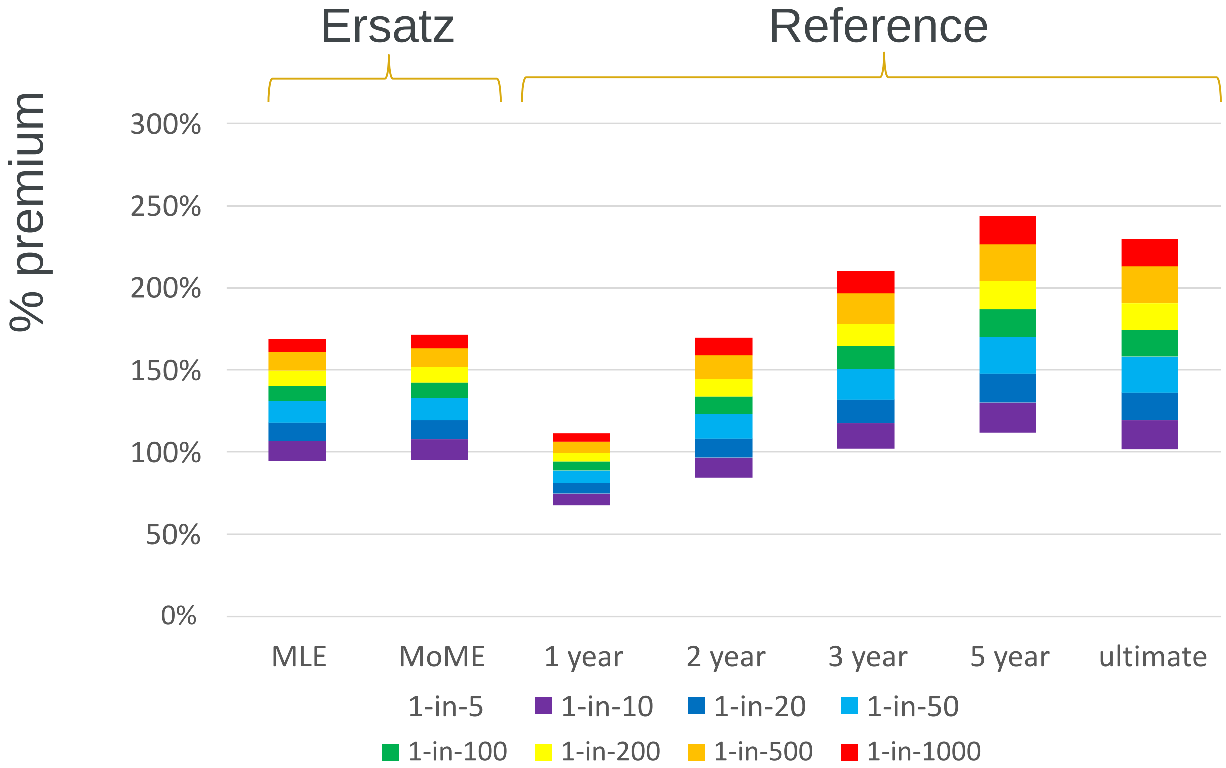
Institute and Faculty of Actuaries

Twenty Possible Futures (from the Reference Model)



Institute and Faculty of Actuaries

Comparing Ersatz to Reference Loss Ratio Percentiles



Institute and Faculty of Actuaries

Comparing Ersatz to Reference Models

- There are several differences:
  - The ersatz models have the same percentiles for all future years, while the reference model has a term structure across future time horizons.
  - The reference model has lognormal rather than gamma distributions, so the tails of the fitted distribution are inevitably too thin.
  - However, at 99.5% the ersatz percentile is sometimes too low and sometimes too high
- Was it foolish to fit a mis-specified model?
  - Yes: if you want to apply statistical criteria such as unbiasedness, consistency and efficiency. These make sense only if the fitted model is well-specified.
  - No: because mis-specification is a fact of life and we need to understand its consequences.

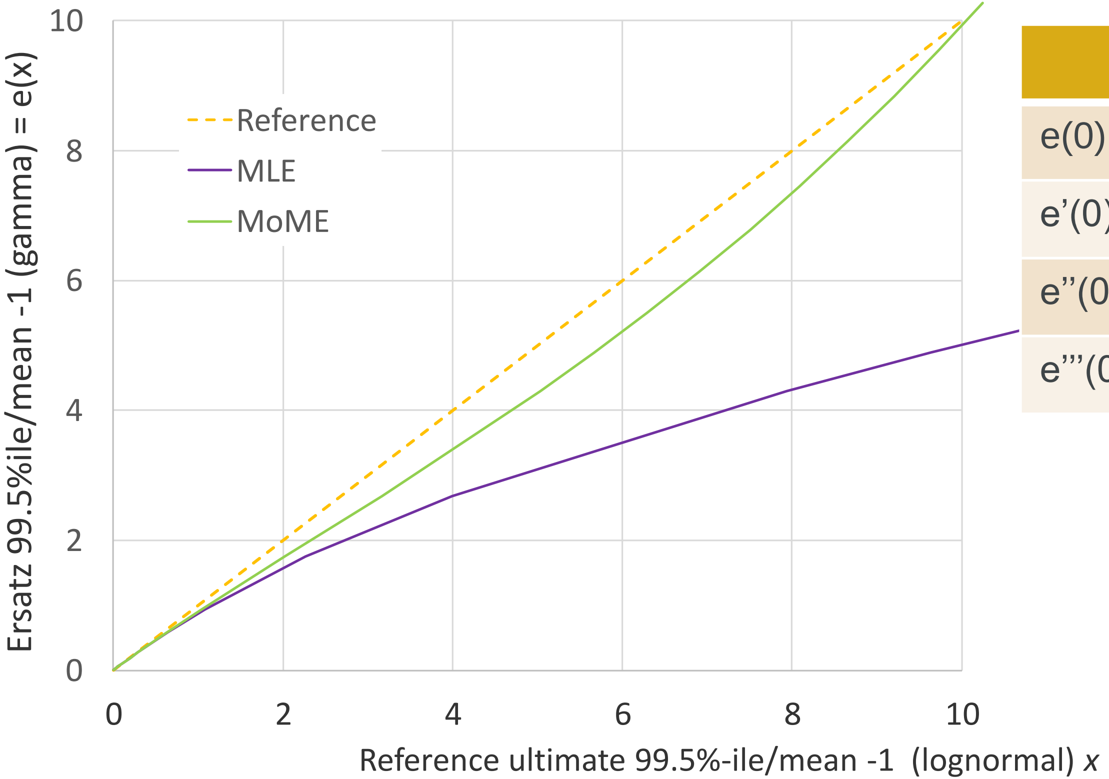
Institute and Faculty of Actuaries

Randomising the Reference Data

- Change TRUE to FALSE in the front sheet.
  - How many exceptions in-sample?
  - How many exceptions out-of-sample?
  - How many in each future year?
- The out-of-sample exception rate is an example of an ersatz model test, because we can perform the test even if the model is mis-specified.
  - We might say a method takes account of model and parameter error if it passes an exceptions test.
- Never forget that the model you are fitting is bound to be wrong, but it just might be an acceptable substitute for the right model.

Institute and Faculty of Actuaries

Theory: What happens for Very Large Data Samples?



	Ref	MLE	MoME
e(0)	0.000	0.000	0.000
e’(0)	1.000	1.000	1.000
e’’(0)	0.000	-0.283	-0.283
e’’’(0)	0.000	0.092	0.359

Institute and Faculty of Actuaries

Conclusions

- Take care over H_0 in goodness of fit tests for fitted (ersatz) models.
  - Don't forget how Lilliefors effects might invalidate your confidence bounds.
- Test ersatz methodologies on randomly generated data.
- Consider asymptotic results for large samples.
- A mis-specified model can still be fit for purpose.
- MoME may be a better estimator than MLE.
  - Depending on how you define your objectives.
- Be explicit about what a good percentile estimate means
  - This involves choices.

Institute and Faculty of Actuaries

Questions

Comments

The views expressed in this presentation are those of invited contributor and not necessarily those of the IFoA. The IFoA do not endorse any of the views stated, nor any claims or representations made in this [publication/presentation] and accept no responsibility or liability to any person for loss or damage suffered as a consequence of their placing reliance upon any view, claim or representation made in this presentation.

The information and expressions of opinion contained in this publication are not intended to be a comprehensive study, nor to provide actuarial advice or advice of any nature and should not be treated as a substitute for specific advice concerning individual situations. On no account may any part of this presentation be reproduced without the written permission of the author.

Institute and Faculty of Actuaries